AD-A045 542 STATE UNIV OF NEW YORK AT BINGHAMTON F/G 14/5
TEXTURE TONE FEATURE EXTRACTION AND ANALYSIS.(U)
AUG 77 S HSU, E KLIMKO F30602-76-C-0211
UNCLASSIFIED RADC-TR-77-279 NL

1 OF 1
ADA045542

END
DATE
FILMED
11-77
DDC

RADC-TR-77-279
Final Technical Report
August 1977

TEXTURE TONE FEATURE EXTRACTION AND ANALYSIS

State University of New York at Binghamton

ROME AIR DEVELOPMENT CENTER
Air Force Systems Command
Griffiss Air Force Base, New York 13441

This report has been reviewed by the RADC Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS it will be releasable to the general public including foreign nations.

This report has been reviewed and is approved for publication.

APPROVED: Donald A. Bush

DONALD A. BUSH
Project Engineer

APPROVED: Howard Davis

HOWARD DAVIS
Technical Director
Intelligence & Reconnaissance Division

FOR THE COMMANDER: John P. Huss

JOHN P. HUSS
Acting Chief, Plans Office

If your address has changed or if you wish to be removed from the RADC mailing list, or if the addressee is no longer employed by your organization, please notify RADC (DAP) Griffiss AFB NY 13441. This will assist us in maintaining a current mailing list.

Do not return this copy. Retain or destroy.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

19

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	18 RADC-TR-77-279
2. GOVT ACCESSION NO.	
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle)	6 TEXTURE TONE FEATURE EXTRACTION AND ANALYSIS.
5. TYPE OF REPORT & PERIOD COVERED	9 Final Technical Report. Apr 76 - May 77
6. PERFORMING ORG. REPORT NUMBER	N/A
7. AUTHOR(s)	10 Dr. Shin-yi Hsu Dr. Eugene Klimko
8. CONTRACT OR GRANT NUMBER(s)	15 F30602-76-C-0211 new
9. PERFORMING ORGANIZATION NAME AND ADDRESS	State University of New York at Binghamton Binghamton NY 13901
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS	62702F 16 62441067 17 10
11. CONTROLLING OFFICE NAME AND ADDRESS	Rome Air Development Center (IRRE) Griffiss AFB NY 13441
12. REPORT DATE	11 August 1977
13. NUMBER OF PAGES	45
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	Same 12 59p.
15. SECURITY CLASS. (of this report)	UNCLASSIFIED
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	N/A

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

Same

18. SUPPLEMENTARY NOTES

RADC Project Engineer: Donald A. Bush (IRRE)

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Texture Analysis
Texture feature extraction
Mahalanobis distance and classifier
Generalized inverse
Stable distributions
Estimation of stable parameters
Digitized B/W photography
Decision map generation
Hit-rate analysis

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A new texture measurement and the Mahalanobis classifier with a generalized inverse scheme ~~have been~~ were developed to generate decision maps of terrain features with digitized B/W photographs on a pixel by pixel basis. Eight scenes within the Northeast test area, four low altitude and four high altitude, ~~have been~~ were analyzed yielding a hit-rate of about 90% with properly digitized image data.

To determine the degree of non-normal behavior of the texture variables, the → next page

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

333 220

4/B

cont. stable distribution models were utilized. Methods of estimating the stable parameters of the texture variables were developed. It is found that fifty % ~~per-cent~~ of the texture variables are not normally distributed. Since the stable distribution models are capable of incorporating the skewness parameters into the classification process, it is recommended as a new classifier for image data analysis.

PREFACE

This report was prepared by Professor Shin-yi Hsu of the State University of New York, Binghamton, New York in partial fulfillment of contract F30602-76-C-0211, for the Rome Air Development Center, Griffiss AFB, New York. The work incorporated in this task consisted of texture-tone analysis, software development, analysis of digitized black and white aerial photography, and estimation of stable parameters of the texture variables. The project was carried out using both the DICIFER Image Processing System of the RADC Image Processing Facility and SUNY-Binghamton Image Data Processing System.

The work described in this report was performed by Dr. Shin-yi Hsu, Principal Investigator, Dr. Eugene Klimko, Faculty Associate, and Graduate Assistants.

This study was performed during the period April, 1976 through May, 1977. Capt. Gregory B. Pavlin and Lt. Cyril Speyrer were the RADC Project Monitors.

LIST OF TABLES

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

LIST OF FIGURES

EVALUATION

This final report covers texture feature extraction by means of measuring the spatial distribution of TONES of the pixels of a given area. Both 1st order and 2nd statistics are used. This effort is under TPO Thrust R2D precision targeting. This effort represents a fine tuning of feature extraction and image classification that will be used in applications to the Automatic Feature Extraction System (AFES) being developed at RADC.

Donald A. Bush

DONALD A. BUSH
Project Engineer

SECTION 1

INTRODUCTION AND SUMMARY

Current image processing capability at RADC employs tonal, spatial, and limited texture feature extractor. To fill the demand for using a powerful feature extractor for real-time object cuing systems, matching pairs of sensed and reference map systems, this study is conducted with two specific objectives: 1) to develop and implement software of texture-tone feature extraction algorithms, and 2) to evaluate these algorithms' potential for object identification and terrain classification using digitized photographic data set. This effort will also provide additional support to a current RADC program with AFATL in the semi-automatic classification of ten terrain types with black/white high altitude photographs.

During the course of this study, Rome Air Development Center (RADC) has provided digitized image data, the DICIFER (Digital Interactive Complex for Image Feature Extraction and Recognition) system for selecting training sets, and the Color Printer for generating color decision maps. The major task of image data processing was conducted at SUNY-Binghamton with the following programs developed for this effort: 1) texture analysis using a (n x n) window size to generate 17 to 23 texture-tone variables for each pixel; 2) the Mahalanobis D^2 logic for classifying pixels into one of the training sets or a reject category, with a generalize-inverse scheme; 3) a step-wise discriminant analysis to select the significant texture variables for the classifier, with a confusion matrix indicating the hit-rate of the training set data; 4) generation of a numerical classification results according to a (10 x 10) cells for a hit-rate analysis; 5) generation of the decision maps using IBM 370 system; 6) manual selection of training sets; and 7) preprocessing capabilities using principal component analysis, and factor analysis.

Eight scenes, four of low altitude and four of high altitude photographs, from the RADC Northeast Test Area (NETA) were used to evaluate the potential of the developed texture analysis algorithms. Terrain types being mapped include metal, pavement, soil, clutivated field, vegetation, water, and composition--a mixture of several categories such as urbanized area. In some

instances, sub-categories are used in the training sets such as two types of pavement, cultivated field and vegetation, respectively.

The results indicate that the developed texture feature extractor together with the Mahalanobis classifier is capable of discriminating the specified terrain types at a high degree of accuracy--a hit-rate of approximately 90% has been obtained using properly digitized photographic data. It is also believed that the hit-rate can still be improved by employing a new classifier which can take into consideration the skewness property of the image data since about 50% of the texture variables are not normally distributed.

The main body of this report will include a literature review of texture analysis, the Mahalanobis classifier, and the analysis of the eight scenes. Preliminary investigation of the potential of the stable distribution theory as a classifier will also be given.

Owing to the operation difficulties of the RADC color printer, only certain decision maps are produced in a color-print format. For analysis purposes, only the numerical classification results based on (10 x 10) cells were utilized to compare against the human interpretation. Hence, the absence of these color decision maps did not impair the hit-rate analysis.

Finally, the principal investigator would like to express his gratitude to Captain Greg Pavlin for his technical assistance performed (for this project) at RADC.

SECTION 2

BACKGROUND

During 1975 and 1976, RADC sponsored a study titled "Digital Image Processing Techniques for Automatic Terrain Classification for Generating Reference Maps From B/W Aerial Photography," conducted by Pattern Analysis & Recognition Corporation, Rome, New York. Since the correct classification rate of this study was about 80% using the feature extractor and the classifier of the DICIFER System, the RADC personnel felt that another study is needed to improve the hit-rate by developing a more powerful texture feature extractor. This led to the current project conducted by Dr. Hsu, using the same data set for the 1975-76 study.

Many factors influence the hit-rate of an image data processing system, the major ones include the performance of the feature extractor and the classifier. The DICIFER System has a limited capability of texture analysis since only six first order statistics are utilized: mean, standard deviation, range, median, high and low. Therefore, to improve the hit-rate, a feature extractor using the second order statistics has to be developed.

The classifier of the DICIFER System employs the Fisher Pairwise logic. It is similar to the conventional classifier based on linear discriminant functions. The hit-rate can be improved further if one employs a classifier whose mathematical assumptions fit the data better than other systems. Hence in this effort a new classifier named the Mahalanobis Logic is developed to accommodate the dispersion characteristics of different training sets. A generalized inverse scheme is also developed to take care of the singularity of the dispersion matrices for each group. In fact, it has been determined that about 30% of the dispersion matrices of the texture variables are singular or near singular, which cannot be inverted under normal conditions.

The hit-rate can also be influenced by the sample size, the location, and the number of the training sets. These problems are of technical ones and will not be discussed in detail in this report. The researchers, however, should be aware of these problems.

In the late 1960s and early 1970s remote sensing researchers found that the spectral data are largely not normally distributed. The conventional

classifiers based on the normal assumptions work only in an empirical sense. The RADC personnel also felt that it is worthwhile to determine the degree of the abnormal behavior of the texture-tone data. The last part of this report is therefore devoted to the discussion of this problem. A new classifier based on stable distribution theory with the normal distribution as a special case will be investigated in the Phase II effort of this study.

SECTION 3

TEXTURE FEATURE EXTRACTION--A REVIEW AND A NEW MEASURE

3.1 A LITERATURE REVIEW

3.1.1 Background

For years, the texture variable has been recognized as one of the important criteria for identifying objects and scenes by the photo-interpreter along with other variables such as tone, size, shape, associated features, etc. Here texture means the apparent minute pattern of detail of a given area, described ordinarily by these terns: smooth, fine, rough, course, and the like. In digital data processing, texture means the spatial distribution of tones of the pixels of a given area. Its attributes have to be specified by the investigator--a specific field of study termed texture feature extraction.

Texture analysis is a rather recent but rapidly growing field of inquiry, though its importance related to visual perception was recognized by Gibson as early as 1950. Over the past twenty years, many texture measures have been proposed. This body of literature has been reviewed by Rosenfeld in 1975. In general, these measures can be grouped into two categories, Fourier-based (power spectrum) features and statistical features. Furthermore, it has been found that statistical features perform much better than the other. (Rosenfeld, 1975).

To obtain texture features, the analyst must specify the size of the window or control area, composed of n x n or n x m pixels, from which texture measures are to be obtained and analyzed. Furthermore, one can classify either only the <u>center point</u> of the window or the whole of the window into one of the specified groups or a reject category. For detailed mapping purposes, the former process is required.

Texture features may include such first-order statistics as mean, standard deviation, range, median, extreme highs and lows. More significant are the second-order statistics, which describe how various pairs of pixels occur in specified spatial relationships.

3.1.2 The Haralick Measure

In the early 1960s, Julesz employed transition probabilities to characterize textures using scanned digital data. Here for any two grey levels i and j, the transition probability p(i,j) measures how often level i and level j occur in horizontally adjacent position. This concept has been followed by many investigators, and expanded to include other directions than horizontal, and pairs of points that are nonadjacent. This is precisely the concept of the spatial dependence matrix introduced by Haralick (1970). Then texture measures are computed from a series of dependence matrices derived from eight scan angles with elements representing relative frequency of tone levels of neighboring cells separated by a predefined distance. The basic texture measures of Haralick's method are the angular second moment (ASM), the angular second moment difference (ASMD), the angular second moment inverse difference (ASMID), and the correlation between neighboring grey tone (COR). Using the directional parameters (0°, 45°, 90°, 135°), one can obtain three measures, namely, average, range, and deviation, from each of the basic measures (ASM, etc). Originally, he proposed to employ 36 texture context features for classification purposes. Since his sample size is small, he selected the following 12 features.

For distance 1: ASM (average, range, deviation),
COR (average, range, deviaiton), and
ASMID (average, range, deviation)

For distance 3: ASM (average)
ASM (range)
ASM (deviation)

The results of his experiment yielded a 70% correct rate using a maximum likelihood classification logic with a normality assumption for the data set. It should be noted that the unit of this analysis is a scene (window), not an individual pixel. The same method was also employed by Dyer et al. in terrain classification with LANDSAT data, yielding a higher hit-rate (about 90%).

3.1.3 Mitchell's Max-Min Descriptor

Recently, Mitchell and Myers proposed a new measure for texture classification based on the human visual system intuition that the important texture information is contained in the relative frequency of local extremes of various sizes in intensity. Thus it is called a max-min texture descriptor. The principal measure here is the number of maxima and minima along a one-dimensional scan direction, under certain threshold conditions. For instance, the maxima is called a maximum only if the intensity falls the threshold amount below the maximum before a higher valued intensity is encountered. Thus, by repeating the process for several threshold settings, analogous to Haralick's distance setting, one obtains a vector of numbers characterizing the textures.

To make this measure invariant to illumination and resolution, Mitchell employed two transformational techniques: 1) taking the log of the intensities first; 2) using the ratio of the number of the extrema at each threshold to the next instead of the extrema themselves.

Compared to Haralick's method, Mitchell's max-min texture analysis performs slightly better, but with much simpler computational effort. Similar to Haralick's method, this analysis uses the whole of the window as a classification unit. It also requires a very large window size to obtain these texture measures. It, therefore, is not applicable to classify <u>individual pixels</u> due to pronounced edge-effect induced by a large window size.

3.2 A NEW MEASUREMENT

To classify individual pixels rather than a group of pixels (windows), it is proposed that a new texture measurement with 17 and 23 variables be derived from (3 x 3, Model I) and (5 x 5, Model II) windows, respectively. In the analysis, the window will move from one pixel to another with an overlapping region between two adjacent pixels; and only the center point is classified.

In Model I, the seventeen texture variables are: (1) through (4) are the four central moments, (5) is the absolute deviation from the mean, (6) is the contrast of the center point from its neighbors, (7) is the mean brightness of the center point relative to its background, (8) is the contrast between adjacent neighbors, (9) is the sum of the squared value of (8), (10) is

the contrast between the second neighbors, (11) is the sum of the squared value of (10), and (12) through (17) are the mean area above and below three datum planes (50, 100, 150). The code names and computational formula of these seventeen variables are given below:

TABLE 3-1. THE TEXTURE-TONE VARIABLES OF MODEL I

Code		Description or Computational Formula
1.	MEAN	average
2.	STD	standard deviation (the four central moments)
3.	SKEW	skewness (the four central moments)
4.	KURT	kurtosis (the four central moments)
5.	MDEVN	$\lvert x_i-\bar{x}\rvert/n$, where x_i = tone value of individual pixel; $\bar{x}$ = mean
6.	MPTCON	$\lvert x_i-x_c\rvert/n$, where x_i = tone value of the center point
7.	MPTREL	$(x_c-x_i)/n$
8.	MINCON	$\lvert x_i-x_j\rvert/n$, i and j are adjacent pixels
9.	MINSQR	$(x_i-x_j)^2/n$
10.	M2NCON	$\lvert x_i-k_k\rvert/n$
11.	M2NSQR	$(x_i-x_k)^2/n$
12.	MADAT1	numerical calculation of mean area above datum 1 (50)
13.	MADAT2	mean area above datum 2 (100)
14.	MADAT3	mean area above datum 3 (150)
15.	MBDAT1	mean area below datum 1 (50)
16.	MBDAT2	mean area below datum 2 (100)
17.	MBDAT3	mean area below datum 3 (150)

In Model II, with a (5 x 5) design, in addition to the above seventeen variables, three measures are extracted to characterize the oscillation nature of wave-forms of the scan lines obtained along both the x and y axes of the data matrix; thus, six variables are available for analysis. They are 1) sum of the contrast values from peak to trough; 2) sum of the distance of peak positions from the origin; and 3) sum of the number of peaks and troughs. This means that there are altogether twenty-three texture variables in Model II.

TABLE 3-2. ADDITIONAL VARIABLES IN MODEL II

Code		Description or Formula
18.	XCONT	(distances from peaks to troughs) along x-axis
19.	XPEAK	(peak positions from the origin) along x-axis
20.	XPANDT	(number of peaks and troughs) along x-axis
21.	YCONT	(distances from peaks to troughs) along y-axis
22.	YPEAK	(peak positions from the origin) along y-axis
23.	YPANDT	(number of peaks and troughs) along y-axis

3.3 COMPARISONS AMONG THE THREE MEASUREMENTS

In sum, we list the above-mentioned three texture measurement in terms of their computational complexity, required window size, and classification unit for comparative analysis.

	Haralick	Mitchell	Hsu
Computational complexity	Rather Complicated	Very simple	Simple
Required window size	Depending on the needed pre-determined distance	Very large	Very small (3 x 3)or (5 x 5)
Classification unit	Only group of pixels being tried; but it is applicable to classify individual pixels	Group of pixels, not applicable to classifying indi-vidual pixels	Pixels
Hit-rate reported	ca 70% (unit of analysis: scene)	ca 80% (unit of analysis: scene)	ca 90% (unit of analysis: pixel)

Owing to classification requirements specified by the AFATL program, the proposed new texture measurement is preferred to Haralick's and Mitchell's methods.

SECTION 4

THE DEVELOPMENT OF THE MAHALANOBIS CLASSIFIER

4.1 BACKGROUND

Over the years, researchers have been using various kinds of strategies to classify image data into meaningful groups. Among them, some have mathematical rigor and others do not. In general, they can be grouped into nonparametric and parametric methods. Examples of nonparametric methods are minimum distance to means, minimum distance to nearest member of a class, etc. Parametric methods used by leading remote sensing centers can be grouped into two broad categories: 1) maximum likelihood ratio decision rules based on a Bayesian formation, a priori probability framework; and 2) a class of linear discriminant functions based on posterior probabilities for classification. (Nalepka, 1970, Swain, 1973). All these classification methods employ training sets (design sets) to define the class characteristics, therefore, they are called "supervised methods." If clustering methods are used to group the populations into distinctive classes, one obtains an "unsupervised" method. Since the unsupervised methods are very time consuming, they are not generally employed and thus will not be discussed here.

4.2 THE GENERAL CLASSIFICATION PRINCIPLE

Within the framework of parametric analysis, one employs a general discriminant analysis to classify an object into one of k types. It is assumed that the spectral/textural signatures of the objects have density functions $P_1(Y), \ldots, P_k(Y)$ where $P_i(Y)$ is the density function for the objects in the ith class. The standard method as given by Rao (1973) is to compute the numerical value of $P_i(Y)$, where Y is the density of the unknown object, for each $i = 1, \ldots, k$, and place the object into class i_o for which $P_{i_o}(Y)$ is largest. In case the $P_{i_o}(Y)$'s are multivariate normal, this method leads to the usual linear discriminant function. The method can also be modified to incorporate a priori distribution π_i, $i = 1, \ldots, k$, if a Bayesian approach is desired. Here, the quantities $\pi_i P_i(Y)$ are computed for $i = 1, \ldots, k$ and the object whose spectral signature is Y is placed in the class which maximizes

$\pi_i P_i(Y)$ for $i = 1, \ldots, k$.

4.3 DIFFERENT APPROACHES

The Bayesian approach has been reported by Fu (1969), and employed by LARS of Purdue University. The linear discriminant function methods are discussed by Morrison (1976). The Fisher Pairwise logic of RADC, and the proposed Mahalanobis logic are examples of the linear discriminant function approaches. Under the same conditions, these three methods should perform equally well. The difference in performance will come from different assumptions that the classifier accepts.

For example, both the Bayesian and linear discriminant function approaches assume that 1) the spectral data are multivariate normal, and 2) they have a common dispersion matrix. Researchers, however, have discovered that spectral data are generally not normal, nor do they have a common dispersion pattern. Once we take into consideration these two problems in the design of the classifier, correct classification rates can be improved substantially.

4.4 THE DEVELOPMENT OF THE MAHALANOBIS CLASSIFIER

Here we describe the classification scheme used in this study. The starting point is the maximum likelihood general principle described in section 4.2. A parametric form of the probability density function is chosen in advance. Usually, this form is a multivariate normal distribution. In this study we first use the normal distribution theory to develop a Mahalanobis classifier and later we introduce the stable classifier in section 6.

After the choice of a parametric model, a training set is used to estimate the parameters in the probability densities for each separate class, e.g., soils, metals, etc. Once the parameters are estimated, the particular individuals may be classified according to the maximum likelihood principle. Each individual is characterized by a vector Y whose coordinates consist of the values of the 17 (or 23 depending on the particular model: 3 x 3 or 5 x 5 being used) variables listed in Table 3-1. Under the normal distribution theory with the non-Bayesian approach, the probability density function for the ith

class is given by the formula

$$(1) \qquad P_i(Y) = \frac{1}{(2\pi)^{P/2}|\Sigma_i|^{1/2}} e^{-1/2(Y-\mu_i)^T \Sigma_i (Y-\mu_i)}$$

where μ_i is the vector of means and Σ_i is the covariance matrix for each class. The maximum likelihood principle then dictates that an unknown object be classified into class i_o if $P_{i_o}(Y) \geq P_j(Y)$ for all j different from i_o. Using logarithms, this rule can be restated as: classify into class i_o if

$$(2) \qquad \log |\Sigma_{i_o}| + (Y-\mu_{i_o})^T \Sigma^{-1}(Y-\mu_{i_o}) \leq$$

$$\log |\Sigma_i| + (Y-\mu_i)^T \Sigma^{-1}(Y-\mu_i)$$

for every i different from i_o. The quantity

$$(3) \qquad D^2 = (Y-\mu_i)^T \Sigma^{-1}(Y-\mu_i)$$

Is called the Mahalanobis distance between the pixel whose variable values are given in the vector Y and the class i whose parameters are μ_i and Σ_i.

Since the values of these parameters μ_i and Σ_i are unknown beforehand, they must be estimated from the data obtained from the training class. Once these estimates are obtained, they are then used for classifying the entire image.

During the initial phases of the study, the assumption that all covariance matrices were equal was made, but quickly discarded in favor of individual covariance matrices for each class. When this was done, the covariance matrices were found to be singular. In this case, the inverse Σ_i^{-1} of the covariance matrix cannot be used, but in its place, the <u>generalized inverse</u> of Σ must be used. In general, all of the classification theory holds if one replaces the true inverse in the formula by its generalized inverse. When the true inverse exists, the algorithm for producing the generalized inverse actually produces the true inverse.

Conceptually, the simplest method for obtaining generalized inverses is to use the spectral decomposition of the covariance matrix

$$(4) \qquad \Sigma = P^T \Lambda P$$

where Λ is a diagonal matrix whose entries are the eigenvalues of Σ and P is a matrix whose columns are the eigenvectors of Σ suitably normalized so that the length of the vector is one. A more efficient algorithm is available for computing generalized inverses. This algorithm is described by Searle (1971), p. 18. It consists principally of solving the systems of linear equations.

$$(5) \qquad \Sigma \Sigma^T X = \Sigma$$

for X and then X will be the required generalized inverse. The generalized inverse is not unique, but any generalized inverse used produces exactly the same classifications. For this reason, we have used the term "the generalized inverse" rather than "a generalized inverse".

4.4.1 Classification Rules

From the discussion in the preceding section, it is clear that the classification rule states that a pixel should be classified into the class type (soil, metal, ...) for which the Mahalanobis distance is smallest which would also be equivalent to putting the pixel into the class for which the posterior probability $P(G|Y)$ is largest. If classification of each pixel is mandatory, then this rule is used. On the other hand, if it is permissible to have some pixels unclassified, then the alternate probability $P(Y|G)$ that an object in group G will have a Mahalanobis distance as big as the observed one for this pixel is found. A cutoff probability is established (generally a small value) and the pixel is declared unclassified if the probability that this pixel belongs to group G is less than the cutoff value. During the study, various values have been used, such as .01, .001, etc. As an example of this rule, suppose that the mandatory classification rule dictates that a pixel whose seventeen measurements are denoted by Y is classified as a metal because metal is the closest class to which this pixel can be identified. However, the probability that a metal pixel will have measurements as

different from the metal class as this particular one is small, say .001. In this case, if unclassified pixels are permitted, then it will be declared unclassified.

4.4.2 Separation of Classes

The most straightforward method for determining whether or not the selected classes (metal, soil, etc.) can be separated is to compute the estimate of the confusion matrix. In computing this estimate one must pay attention to Foley's principle of sample size. In particular, the number of samples selected for each class must be at least three times the number of measurements associated with each pixel. (One can actually use the rank of the covariance matrix for the measurements in case there are multiple collinearities in the data. This would reduce the sample size somewhat, although sample size was not a problem in this study.)

The estimate of this matrix is simply an array of the percentages of cases misclassified into each of the classes. If all of the cases are correctly classified, then separation is perfect. In this case, it is probably possible to reduce the number of measurements used at each pixel.

Stepwise discriminant analysis can be utilized to select measurements or features which are most useful to discriminate between classes. This stepwise procedure consists of selecting the features--one at a time--which contribute most toward the separation of groups. The selection procedure can be stopped as soon as enough features have been selected to produce a complete separation of the groups. In the stepwise discriminant procedure, an F test based on the likelihood ratio criteria is made to select the features, rather than an analysis of the confusion matrices.

4.4.3 Ordination Procedures

At each pixel, a set of features or measurements are taken which describe the texture and tone of that particular pixel. During the initial phases of the study, the major objective was to select the features which would contribute most to the separation of the training classes. Initially a large number of texture features were chosen. Principal components and related factor analysis methods were used to determine the number of non zero eigenvalues in the covariance matrix of the features. This information describes the number of essentially distinct features which exists within the data.

SECTION 5

THE ANALYSIS

5.1 THE DATA SET

The data set for this study is composed of eight scenes of the RADC's Northeast Test Area: Griffiss AFB, New York (GALA, GAHA); Verona, New York POL Storage (VPLA, VPHA); Stockbridge, New York, SAM Site (SBLA, SBHA); and Utica, New York, Rail Yards (URLA, URHA) at both low altitude (LA), and high altitude (HA). Their geographic locations, elevations and flight height are given in Table 5-1.

TABLE 5-1. THE DATA SET*

Scene	Geographic Coordinates	Elevation	Image Flight Height
1 GALA	43°14'N, 75°25'W	515'	15,500'
GAHA			61,500'
2 VPLA	43°08'N, 75°36'W	500'	15,500'
VPHA			60,500'
3 SBLA	43°02'N, 75°39'W	1290'	16,000'
SBHA			60,500'
4 URLA	43°07'N, 75°13'W	410'	15,400'
URHA			60,500'

*RADC - TR - 76 - 196 Final Report by PAR, pp. 8-9.

After digitization the ground resolution of the low altitude and high altitude images are approximately 8.75 feet, and 56.75 feet, respectively. It should be noted that the images have a much higher resolution level. Stored on tapes, each scene is then composed of (256 x 256) pixels, with tonal densities ranging from 0 (black) to 255 (white).

For hit-rate analysis, high resolution photographs are provided by RADC as the basis of the ground truth information to be obtained by manual photo-interpretation.

5.2 THE HARDWARE FACILITY

To carry out this study, the DICIFER system at RADC, and the IBM 370-158 general purpose computer at SUNY-Binghamton were utilized. While the RADC hardware system was used to 1) digitize the image data, 2) store the data in computer compatible tape, 3) select the initial training sets, and 4) generate the color decision maps, the IBM 370 system was used mainly to process the data with the software developed at Binghamton.

The IBM system was also employed to generate the tone maps and the final decision maps with the printer. This allows the researcher to select the appropriate training sets manually for the classifier. The decision map was translated in a numerical classification of each group according to (10 x 10) cells which was then used to check against the manual interpretation result for a hit-rate analysis.

5.3 THE SOFTWARE SYSTEM

The computer programs used in processing the image data include the following capabilities:

(1) Texture-tone analysis using a (n x n) window size to generate 17 to 23 texture variables for each pixel.

(2) The Mahalanobis Logic for classifying pixels into one of the design sets or a reject category.

(3) A generalize-inverse scheme to invert singular or near-singular matrices.

(4) Generation of a numerical classification results according to a (10 x 10) cells as the basis of hit-rate analysis.

(5) Stepwise discriminant analysis to select significant texture variables.

(6) Pre-processing capability including principal components, factor analysis, etc.

(7) Generation of decision maps using IBM 370 system.

(8) Generation confusion matrix with the training set data.

(9) Generation of D^2-distances with probability levels between classes.

(10) Manual selection of training sets.

5.4 GENERATION OF A DECISION MAP

Decision maps are produced using the following computational steps:

(1) Generate the texture-tone variables for the training sets and the unclassified set.

(2) Compute the parameters for the discriminant function.

(3) Classify the training sets, thereby obtaining the confusion matrix.

(4) Classify the unknown set.

The step one is done by the program "GENVAR." This program reads control cards specifying the position and size of each training set. The variables are computed as documented in the GENVAR program and written onto a disk data set.

The remaining steps are done by program "SPCMAP." It reads control cards instructing it as to where the training sets and test set may be found, and how many points are contained in each. It also reads titles for the training sets and lists of symbols to be used for the map output. For each training set, the centroid vector and covariance matrix is computed. The covariance matrices are inverted using a generalized inverse scheme. In order to classify a point, the quadratic forms of the differences of the point vector and each group centroid over the corresponding group inverse covariance matrices are computed. This quadratic form is the Mahalanobis D^2. The group which is closest to the given point is chosen.

If the user wishes, a map may be generated in which points that do not strongly belong to any training set are excluded or classified as rejects. This is possible because D^2 is Chi-square random variable having a probability value. If its probability is below a certain fixed cutoff, the point is rejected. Rejects are left blank on the decision maps.

5.5 HIT-RATE ANALYSIS

To assess the performance of the developed texture measures, hit-rate analyses of the test sites have been carried out. The procedures include, 1) placing a (10 x 10) grid onto both the computer decision map and the photo print of the test area, and 2) estimating and enumerating the percentage of all terrain type classes in each cell. The hit-rate is computed as:

$$\text{Hit-rate} = 1 - \frac{\text{Difference between photo-interpretation and computer-decision map}}{\text{photo-interpretation (in terms of total area of each class)}}$$

$$\text{or} \qquad = 1 - \text{error-rate}$$

The following table summarizes the results of the analyses. It can be concluded that for a larger area a hit-rate of about 90% can be obtained with properly digitized images. The hit-rate for small areas is statistically meaningless. It has been found that digitizing errors exist in the high altitude images (GAHA, VPHA, URHA), thus hit-rates for these frames must be obtained by using sub-groups within categories. For instance, in VPHA two types of cultivated fields were used in the training sets.

TABLE 5-2. HIT-RATES

	Vegetation	Cultivated Field	Metal	Soil	Pavement	Water	Composition
GALA	88.4%	98.46%	90%	53.13%	92.28%	--	--
SBLA	89.81%	82.59%	(in rejects)	87.13%	--	Too small an area for meaningful assessment.	
URLA	(in rejects)	--	80.50%	45.90%	85.24%	--	87.40%
VPLA	90.00%	85.5%	95.0%	86.00%			
GAHA*	(Veg. & Cul. field)	88.51%	--	85.53%	72%	--	75.9%
SBHA	99%	95%	--	98%	--	--	95%
VPHA** (5 x 5)	60%	84.1%	--	93.75%	70.1%	--	85.10%
URHA***							

*Uneven densities for cultivated field, NE corner vs. SW corner. Thus vegetation and cultivated fields are treated as one group.

**Uneven tones for cultivated field due to "digitizing error" which induced confusion between vegetation and cultivated fields. (Top one-third vs. lower two-thirds.)

***No meaningful hit-rates can be obtained due to "digitizing error."

5.5.1 GALA

The analysis shows that a hit-rate over 90% (except for soil) has been achieved by Model I. It should be noted that the photo-interpretation of the ground truth is obtained from a high resolution aerial photo rather than low resolution images from which the computer decision map was derived. The author has investigated further the problem regarding the soil class using the output from Model II. It was first thought to be the "edge effect." However, since the mis-classification of the soil pixels was largely eliminated in Model II, it was therefore determined to be "resolution effect," which was purposely induced into the images during the process of digitization. The performance of Model II is better than Model I, except it has a larger area of reject, and occasionally pronounced edge effect.

5.5.2 SBLA

In general, the overall terrain pattern came out very well in the decision map. The SAM site and tanks (metal-objects) were correctly identified using the reject category.

Similar to GALA, "resolution" effect occured at the "edge" of two distinctive classes, and at certain vegetation areas.

The rejects region were about 10% of the total area. There was no significant difference between the "reject" pattern determined by P(X/G) = 0.01 and that by P(X/G) = 0.001. This means that the pixels being rejected were really different from the design sets.

5.5.3 VPLA

The overall terrain pattern in the decision map was good in the sense that essential types were correctly identified. In terms of a detailed hit-rate analysis, the correct classification rate is about 85% (excepting pavement). Two factors caused the error rate: 1) asphalt-paved road could not be differentiated from fields used for recreational purposes; and 2) a new concrete road was being built at the time the image was taken--many types of "pavement" were present at this section of the image. If cultivated field and pavement were treated as one group, the hit-rate will be over 95%.

To achieve a correct classification of this frame, four types of cultivated field were used in the training sets to cover significant local

variations. In terms of the training set itself, a hit-rate of 98.4 was achieved. However, in terms of the test set, the hit-rate is much lower due to significant local variations.

5.5.4 URLA

The URLA was a more complicated frame, thus an iterative process was utilized to generate the decision maps. The more obvious classes, such as metal, pavement, composition, etc., were processed first and the "uncertain" and insignificant (in terms of aereal coverage) vegetation, were left out. The "reject" area thus represents mixed water, vegetation and cultivated fields, etc. At both 0.01 and 0.001 probability reject levels, the area showing "rejects" is very small, corresponding to a potential area of mixed water and vegetation.

5.5.5 SBHA

This was the only frame in the high altitude image group that had few digitization problems. The generation of the decision map was therefore rather straightforward due to less complexity in the terrain configuration, a very high hit-rate was achieved (over 95%).

5.5.6 VPHA

Image digitization error existed in the frame; specifically the upper one-third is much lighter than the lower two-thirds portion of the frame. Using the RADC DICIFER system, it was determined that a 30-point difference existed between these two portions of the frame for cultivated field category.

5.5.7 GAHA and URHA

The same digitization problem caused the fact that the NE corner of GAHA is much lighter than the same terrain types in the SW corner. To process this frame, two artificial types of cultivated fields had to be used in the design sets. Since vegetation and cultivated field classes were really confused by this digitization effect, they were grouped as one class in the hit-rate analysis.

We were unable to obtain a reliable hit-rate for URHA due to the same digitization problem. However, we were able to produce a fairly good deci-

sion map in terms of the overall terrain pattern.

5.6 A GENERAL COMMENT ON THE DECISION MAP MAKING

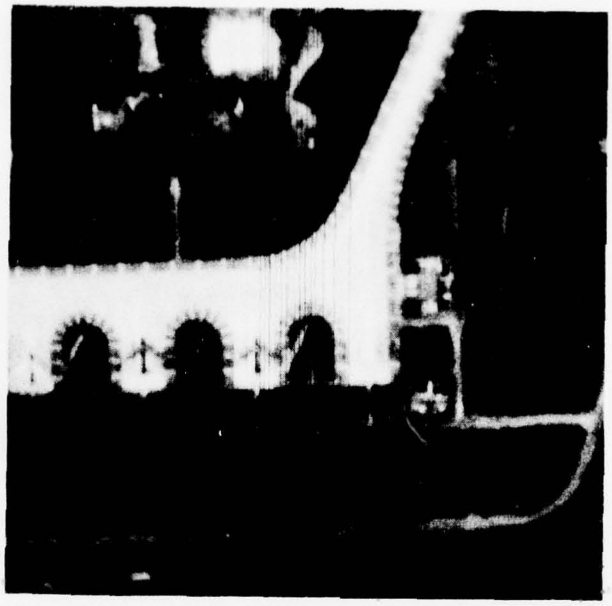
In addition to the feature extractor and the classifier, the hit-rate and false alarm rate also depend on the factors regarding sample size, the location, and the number of the training sets.

The minimum sample size problem has been investigated by Forley. His principle states that for a valid analysis the minimum sample size is three times as large as the number of the variables used. For instance, if one employs ten texture variables in the analysis, the minimum number of each training set is 30. It is also our experience that the Forley principle is valid and that empirically, the sample size of each training set should be greater than 30 pixels in general.

Improper training sets generally lead to a low hit-rate. To avoid such an error, one should first employ the <u>confusion matrix</u> (from the training sets), to identify confused classes, and to locate mis-classified pixels on the (preliminary) decision map. Then, one should change the location of the training sets in order that "pure" training sets can be obtained. This is an iterative process, and it can be done manually or by the operator using the interactive graphics, namely, using a cursor on the color monitor with a terminal control. Once the correct classification rate in the design set reaches a level of 90% or over, the investigator can proceed to classify the test set data.

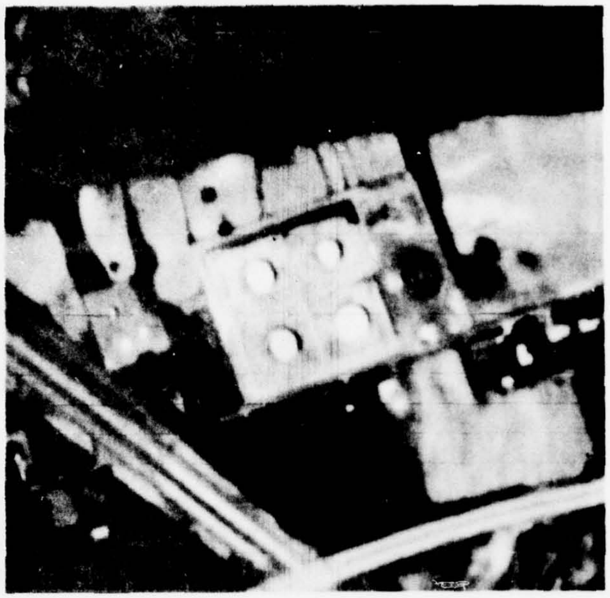
To classify the test set, one can classify each group at a time, or classify many groups in one process. Theoretically, the first method will yield a lower hit-rate because there is only one probability value for each pixel to be used in the classification, which may not be maximum once other groups are introduced. Most likely, this method will produce overlapping groups, that is, an individual pixel may belong to several groups.

To assure that the test sets are properly classified, all the desired groups should be introduced in the design set. Furthermore, if local differences exist within one group, sub-groups should be introduced. These subgroups can be labeled as one group only after the decision map is produced. It is our experience that a sufficient number of groups should be used in the design sets; otherwise, mis-classifications or rejects will be substantial.

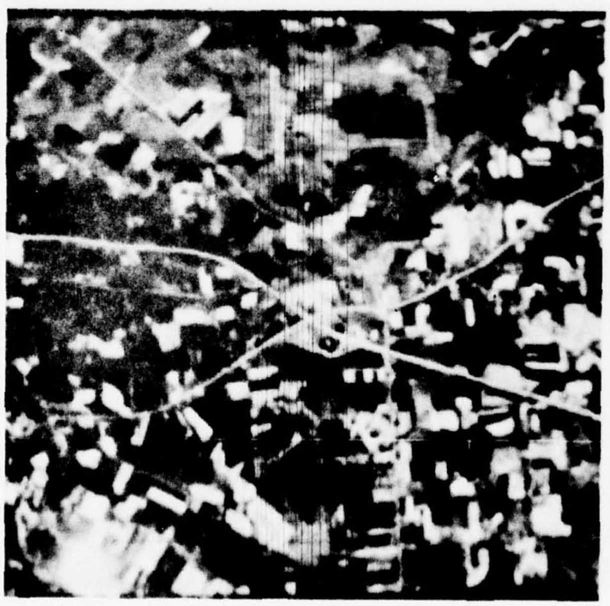


GALA--ORIGINAL DIGITIZED PHOTO

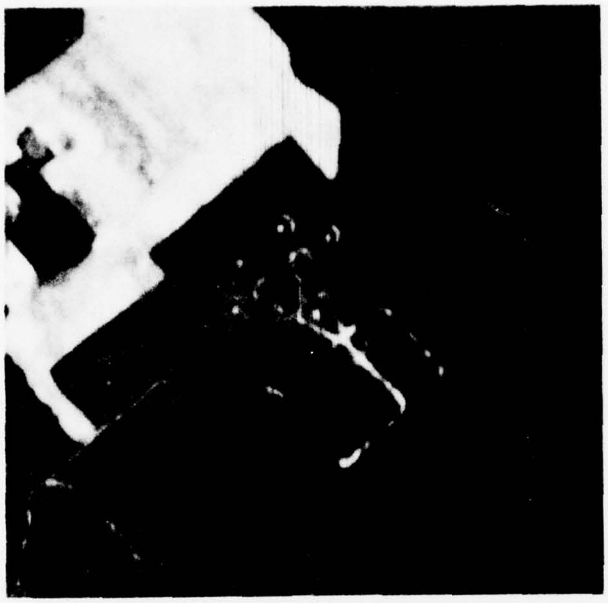
GAHA--ORIGINAL DIGITIZED PHOTO



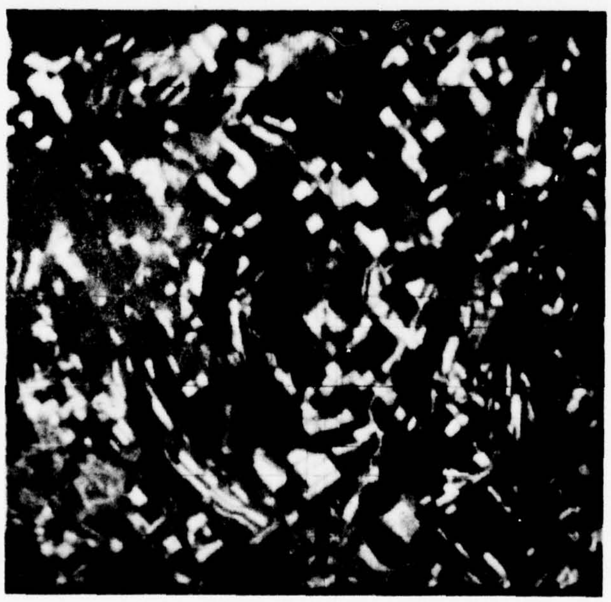
VPLA--ORIGINAL DIGITIZED PHOTO



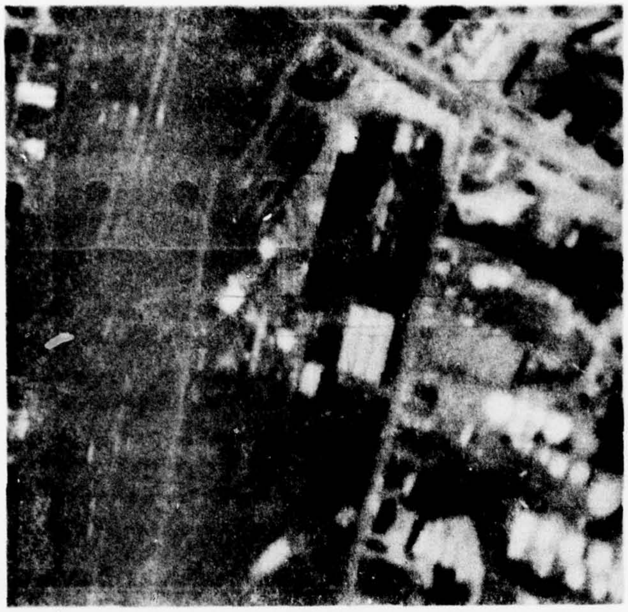
VPHA--ORIGINAL DIGITIZED PHOTO



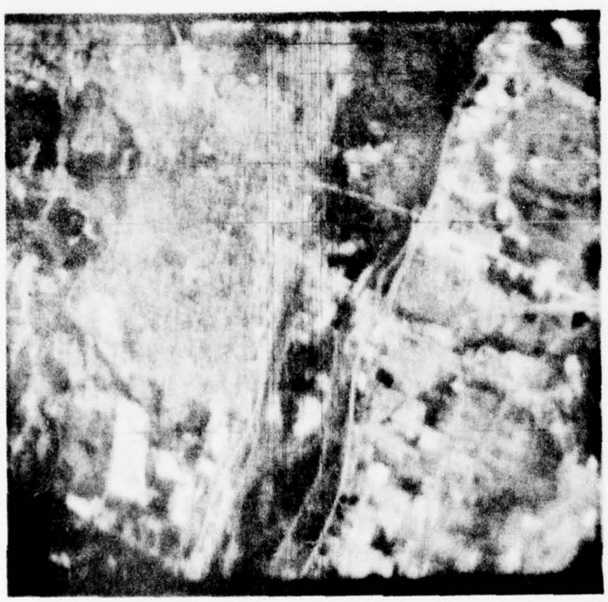
SBLA--ORIGINAL DIGITIZED PHOTO



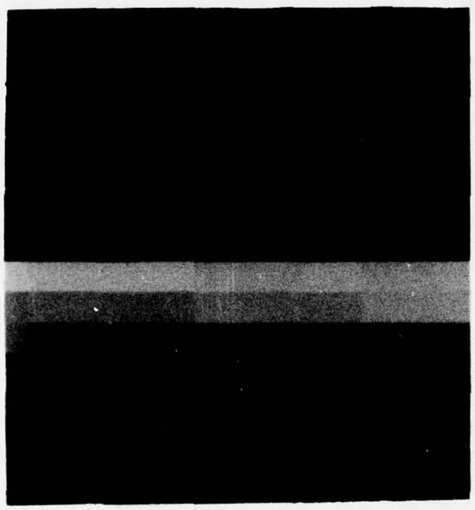
SBHA--ORIGINAL DIGITIZED PHOTO



URLA--ORIGINAL DIGITIZED PHOTO

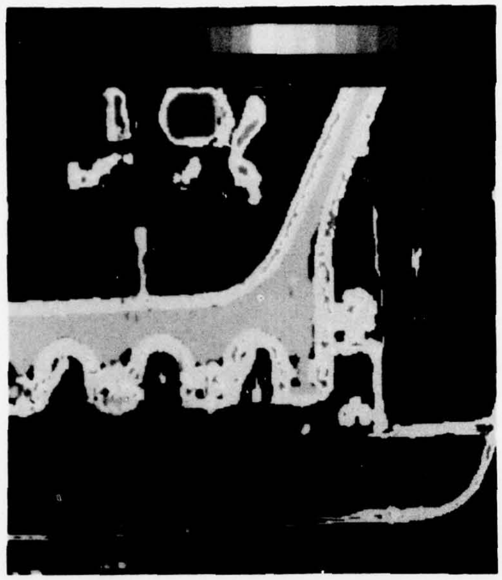


URHA--ORIGINAL DIGITIZED PHOTO

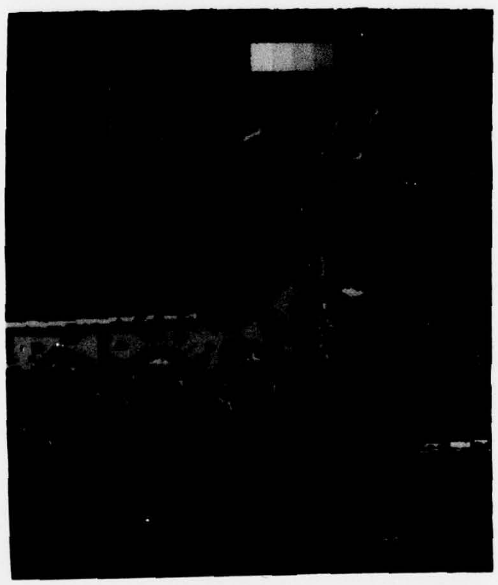


Should be interpreted
according to the
original data sets

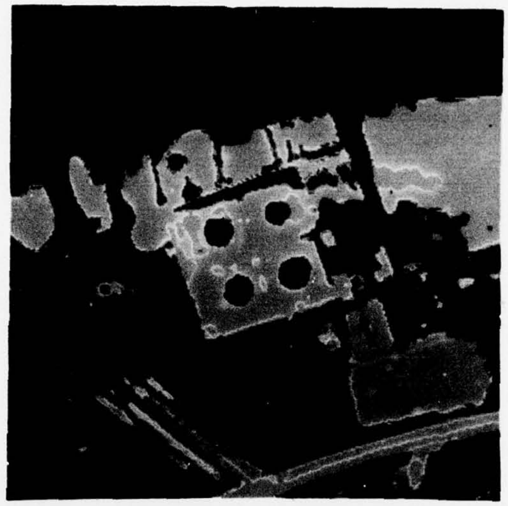
The Color Codes for the Decision Maps



DECISION MAP: GALA (3 X 3) WITH NO REJECTS

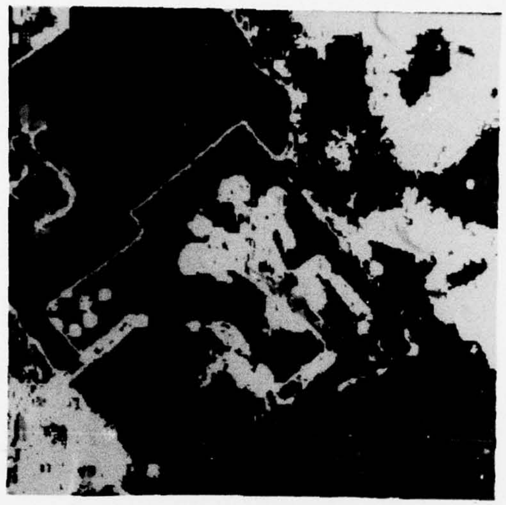


DECISION MAP: GALA (3 X 3) WITH REJECTS (DEEP BLUE)

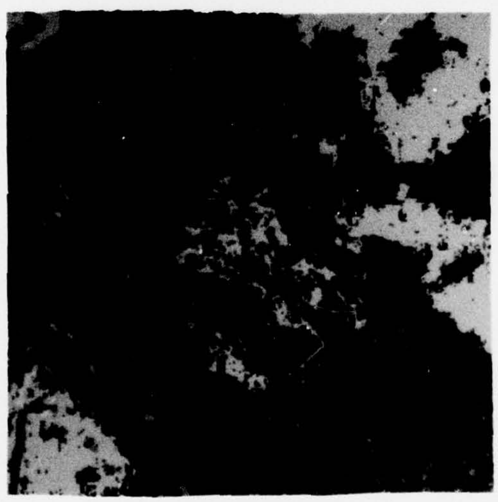


DECISION MAP: VPLA (3 X 3) WITH NO REJECTS

DECISION MAP: VPLA (3 X 3) WITH NO REJECTS
NOTICE THE EDGE-EFFECT

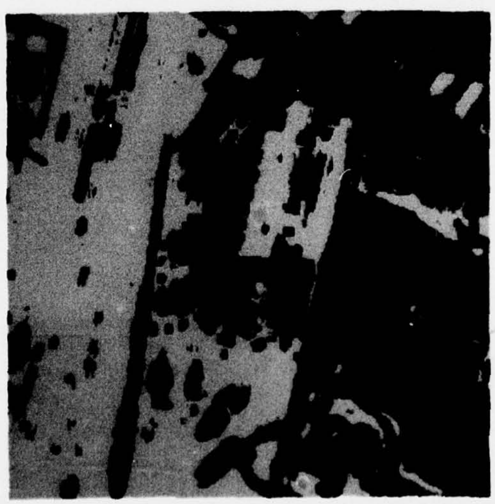


DECISION MAP: SBLA (3 X 3) WITH NO REJECTS



DECISION MAP: SBLA (3 X 3) WITH REJECTS (DEEP BROWN)

DECISION MAP: URLA (3 X 3) WITH NO REJECTS



DECISION MAP: URLA (5 X 5) WITH NO REJECTS

SECTION 6

ESTIMATION OF STABLE PARAMETERS

6.1 CHARACTERISTICS OF STABLE DISTRIBUTIONS

As part of this study, stable distributions were considered as alternatives to the multivariate normal distributions on which the Mahalanobis classifier is based.

Stable distributions are best defined in terms of their characteristic functions $\phi(t)$ or its logarithm which in the univariate case (single feature) is given by:

$$\log_e \phi(t) = \log_e \int_{-\infty}^{\infty} e^{itx}\, dF(x)$$

(1)

$$= it\delta - \gamma|t|^{\alpha} [1 + i\beta \frac{t}{|t|}\omega(t,\alpha)]$$

where

$$\omega(t,\alpha) = \mathrm{Tan}(\pi\alpha/2), \text{ if } \alpha \neq 1$$

$$= \frac{2}{\pi} \log |t|, \text{ if } \alpha = 1$$

and δ is a location parameter, $\gamma \geq o$ is a scale parameter, α is the characteristic and β is the symmetry parameter. The parameter δ plays the role of the mean and is equal to the mean whenever the mean exists. The variance is always infinite when $\alpha < 2$, however the parameter γ plays the role of a scale parameter and β is the symmetry parameter. In particular, if $\beta = o$, the distribution is symmetric about δ. The parameter α is called the characteristic of the distribution and $o < \alpha \leq 2$. If $\alpha = 2$, then

$$\log \phi(t) = it\delta - \gamma t^2$$

which is the characteristic function of a univariate normal distribution with mean δ and variance $\gamma/2$. If $\alpha = 1$, the distribution is Cauchy. For all other values of α, the density exists but a closed formula for it is not known. Various power series expansions for the density exist which may be found for example in DuMouchel (1971) and Feller (1966).

The most important parameter for a stable distribution is α because it

determines the type of the distribution. When $\alpha = 2$, the distribution is normal. In this study, the estimation of α is used to assess the normality or lack of normality of the data. If the estimate for α is close to 2, then the data may be assumed to be normal. The results of this study indicate that many of the features have distributions which are not normal.

The main reason for considering stable distributions are given here. The first advantage of stable distributions lies principally in the generalized central limit theorem. Among other things, it states that if X_1, X_2, ..., X_n are independent identically distributed random variables having any distribution with finite variance, the distribution of their sum will tend toward the normal distribution as n increases. This is an exceedingly powerful result as it shows that if an observable random variable is produced as the sum of many independent nearly identically distributed random variables, its distribution will be approximately normal, no matter what the distribution of the underlying variables. If the variance is not finite, a limiting distribution for the sum may still exist. The vital point is that if it does exist, it is a stable distribution. Every member of the stable family is such a limit, and no distribution other than a stable distribution may be such a limit. This unique property gives stable distributions an important position in statistical theory and practice.

One more property of stable distributions which does not have the theoretical impact of the generalized central limit theorem, but nevertheless makes them valuable as models of empirical results, is the following: Experimental image data by no means need be normal. Mixed in with a bulk of roughly normal observations may be one or two outliers. A whole body of literature has accumulated on what to do with them. The question usually asked is whether to keep them as valid measurements which will admittedly grossly affect the results, or discard them as noise. The principal problem is that most widely available statistical tests are incapable of properly handling empirical distributions in which the sum of a set of random variables is largely dominated by one of the observations. For this reason, the outliers are usually discarded. The method of choice would seem to be to keep all the data, but use a method of analysis which is capable of dealing fairly with such distributions. Recent experimentation in the field of

economics by Mandelbrot (1963) has indicated that stable methods are well suited to this task. Feller (1966) also presents a small but interesting survey of physical processes governed by stable laws.

One of the important properties of multivariate normal distributions is that every linear combination of its components has a univariate normal distribution. This property also carries over to stable distributions. That is, every linear combination of the components of a multivariate stable distribution has a univariate stable distribution.

Another important property of stable distributions is that the sum of two independent stable variates with the same characteristic α is itself stable with the same characteristic α as the summands.

A final advantage to modelling by means of stable distributions is that skewed distributions can be accommodated.

There is a theory of multivariate stable distributions which is similar to the theory of multivariate normal distributions and in fact contains the normal theory as a special case. As in the univariate case, the multivariate stable distribution is best described by its characteristic function. Details of the multivariate stable distributions are found in Press (1972).

6.2 ESTIMATION METHODS

Various methods for estimation of stable parameters have been proposed in the statistical literature. During this study, these methods have been evaluated for their practical value. Some methods have been found to be reasonably useful while others are useless.

For symmetric distributions, a relatively easy method for computing estimates of the parameters α, δ, and γ are given by Fama and Roll (1968, 1971). They have shown that truncated means are good (and, obviously, unbiased) estimators of the location δ. The degree of truncation which provides minimum error variance is a function of α, but using the central fifty percent gives quite good results. They also show that $\hat{c} = .605(X_{.72}-X_{.28})$, where X_p is the estimate of the P^{th} fractile of the sample, is a reasonable estimator of the scale. It has small asymptotic bias. Its error variance, though small enough for non-critical work, is significantly larger than the Rao-Cramèr lower bound. They also investigate the approximation of α by

choosing $\hat{\alpha}$ such that, for some previously specified fractile P, the theoretical fractile of $s_{\hat{\alpha},0}$ is the same as the sample fractile. Again, the optimal value for P varies with the true alpha and the sample size. These estimates of alpha showed some small bias and an error variance which is considerably larger than could be had. However, all three estimators are excellent when one considers their elegant simplicity.

DuMouchel (1971) did some work on maximum likelihood estimation of stable parameters. The data is censored into discrete classes. The likelihood function is evaluated for trial parameters using asymptotic formulas for extreme values and the fast Fourier transform of the characteristic function for central values. This method has two drawbacks. First, the censoring of the data causes some information loss. Second, and most serious, is its incredible slowness. Each iterative try for a better parameter estimate requires that the characteristic function be evaluated at many points, and several Fourier transforms be done.

Other methods have been proposed based on characteristic functions. These methods in general have been shown unworkable. The method of maximum likelihood is the best method for obtaining estimates in the case of stable distributions.

At this time there are no known methods for explicitly evaluating stable densities and distribution functions to any realistic precision rapidly enough to be feasible for production work. Therefore, it has been the goal of this study to investigate algorithms which are quite slow but accurate. These then are used to generate tables suitable for interpolation. Means of storing and accessing these tables for the evaluation of stable densities and distributions at high speed have also been developed.

All of the rather complex details can be found in Masters (1977). A large number of Monte Carlo experiments were run in order to test the algorithm for computing these parameter estimates. All of the estimaters showed very small bias and standard error for most values of α. Some problems were noted when α is close to 1 or 2. Fortunately, values of α close to 1 do not seem to occur. For $\alpha = 2$, one can use estimators for normal distributions.

6.3 PRELIMINARY RESULTS-NON NORMAL BEHAVIOR OF THE TEXTURE VARIABLES

In order to assess the normality assumption for the texture variables, an estimate of the stable distribution parameters was made for the data available from the scenes VPLA, VPHA, SBLA and GAHA. Some results of these estimates are presented in Table 6-1. The complete set of seventeen texture variables is given for four separate classes: pavement, vegetation, cultivated fields (1) and soil. The estimated value of α is given in the table. The largest value of α given by the estimation algorithm is 1.99, and the smallest is 1.01. Thus a value of 1.99 means that the estimate is between 1.99 and 2.00. A value of 1.99 indicates that the data are normally distributed whereas smaller values indicate that stable models are more appropriate. The complete set of values run is too large to present. A total of 476 different variables have been estimated over the four photographs. Of these, 263 had α values different from 1.99. This means that 55% of the texture variables are not normally distributed.

A complete set of estimates for the β parameter is not available, but preliminary estimates indicate that β does deviate from 0 (symmetric distributions).

These results indicate that development of a stable classifier will produce good results.

TABLE 6-1. VALUES OF α FOR VPHA

Texture Variable	Pavement	Vegetation	C. Field 1	Soil
1. MEAN	1.20	1.99	1.99	1.15
2. STD	1.59	1.20	1.71	1.99
3. SKEW	1.31	1.99	1.99	1.99
4. KURT	1.01	1.53	1.58	1.45
5. MDEVN	1.99	1.21	1.69	1.99
6. MPTCON	1.18	1.35	1.69	1.01
7. MPTREL	1.99	1.99	1.84	1.08
8. MINCON	1.24	1.22	1.55	1.41
9. MINSQR	1.27	1.21	1.64	1.99
10. M2CON	1.99	1.26	1.63	1.50
11. M2NSQR	1.99	1.24	1.78	1.66
12. MADAT1	1.18	1.99	1.99	1.15
13. MADAT2	1.15	1.01	1.01	1.18
14. MADAT3	1.01	1.01	1.01	1.99
15. MBDAT1	1.01	1.99	1.01	1.01
16. MBDAT2	1.85	1.99	1.99	1.01
17. MBDAT3	1.34	1.99	1.99	1.01

REFERENCES

1. DuMouchel, W. H. Stable Distributions in Statistical Inference. Ph.D. Dissertation, Yale University, 1971.

2. DuMouchel, W. H., "On the Asymtotic Normality of the Maximum Likelihood Estimate When Sampling from a Stable Distribution," The Annals of Statistics, 1973, Vol. 1, No. 5, pp. 948-957.

3. Dyer, C. R., et al. Experiments in Terrain Classification on LANDSAT imagery by Texture Analysis. Computer Science Technical Report Series (TR-383 ENG 74-22006). University of Maryland College Park, Md., 1975.

4. Fama, E. F. and Roll, R., "Some Properties of Symmetric Stable Distributions," Journal of the American Statistical Association, Vol. 63 (1968), pp. 817-836.

5. ______________________, "Parameter Estimates for Symmetric Stable Distributions," Journal of the American Statistical Association, Vol. 66 (1971), pp. 331-338.

6. Feller, W. An Introduction to Probability and Its Applications, Vol. II, New York, Wiley (1966).

7. Foley, D., "The Probability of Error in the Design Set as a Function of the Sample Size and Dimensionality," Technical Report RADC-TR-71-171, (AD 738648), 1971.

8. Fu, K. S., D. A. Landgrebe, and T. L. Phillips, "Information Processing of Remote Sensical Agricultural Data," Proceedings of IEEE, Vol. 57, No. 4 (April 1969), pp. 639-653.

9. Haralick, R. M., "Texture Tone Study With Applications to Digitized Imagery," Technical Report 181-1, Center for Research, Inc., University of Kansas, Lawrence, Kansas, 1970.

10. Julesz, B., "Visual Pattern Discrimination," IRE Trans. on Information Theory, IT-8, 1962, pp. 84-92.

11. Mandelbrot, B, "New Methods in Statistical Economics," The Journal of Political Economy,LXXI, pp. 421-440, 1963.

12. Masters, T., "Estimation of Parameters for Univariate Stable Distributions," Masters Thesis, State University of New York at Binghamton, 1977.

13. Morrison, D. F., Multivariate Statistical Methods, New York: McGraw-Hill, (2nd edition), 1976.

14. Nalepka, R., "Investigation of Multispectral Discrimination Techniques," Report No. 2264-12-F Willow Run Laboratories, Institute of Science and Technologies, University of Michigan, 1970.

15. Press, S. J., "Multivariate Stable Distributions," Journal of Multivariate Analysis, Vol. 2, 1972, pp. 444-462.

16. ___________, "Estimation in Univariate and Multivariate Stable Distributions," Journal of the American Statistical Association, Vol. 67, 1972 (a), pp. 842-846.

17. ___________, Applied Multivariate Analysis. New York, Holt Rienhart and Winston, Inc., 1972(b).

18. RADC/PAR Digital Image Processing Techniques for Automatic Terrain Classification for Generating Reference Maps for B/W Aerial Photography, RADC, 1976.

19. Rao, C. R. Linear Statistical Inference and Applications (2nd edition). New York, Wiley, 1973.

20. Rosenfeld, "Visual Texture Analysis: An Overview," Computer Science Technical Report Series (TR-406. F44620-72C-0062), University of Maryland, College Park, Md., 1975.

21. Searle, S. R., Linear Models, Wiley, New York, 1971.

22. Swain, P. H., "Pattern Recognition: A Basis for Remote Sensing Data Analysis," Laboratory for Applications of Remote Sensing, Purdue University, W. Lafayette, Ind., LARS Note 111572.

MISSION
of
Rome Air Development Center

RADC plans and conducts research, exploratory and advanced development programs in command, control, and communications (C^3) activities, and in the C^3 areas of information sciences and intelligence. The principal technical mission areas are communications, electromagnetic guidance and control, surveillance of ground and aerospace objects, intelligence data collection and handling, information system technology, ionospheric propagation, solid state sciences, microwave physics and electronic reliability, maintainability and compatibility.

AMERICAN REVOLUTION BICENTENNIAL 1776-1976

Printed by
United States Air Force
Hanscom AFB, Mass. 01731